

Note on Hong and Sommese's conjectures on Critical Points of Cyclotomic Polynomials

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Abstract

In this note we provide the results of computational checks on the various conjectures of Hong and Sommese's "Number of Real Critical Points of Cyclotomic Polynomials." We find a family of counterexamples to conjecture 3[3] and provide a strengthened form of conjecture 3[2] suggested by our numerical search.

1 Introduction

We place some examples below

First, we reintroduce some of their notation. For an odd squarefree integer $m > 1$, the authors define N_m to be the number of zeroes of the derivative of the m th cyclotomic polynomial. We define Drop to refer to a decrease in the number of roots Φ'_m , Hong and Sommese conjectured that, for an odd squarefree integer n and primes p, p' , $p < p'$, $p, p' \nmid n$, the following statement holds(Conjecture 3[3]):

$$N_{np} \leq N_{np'}$$

However, in our numerical search, we found a large variety counterexamples to this conjecture. We provide some interesting examples below.

Base n	p	p'	N_{np}	$N_{np'}$	Drop
$3 \cdot 59 = 177$	5	7	7	6	2
$3 \cdot 67 \cdot 71 = 14271$	5	11	11	7	4
$3 \cdot 5 \cdot 7 \cdot 61 = 6405$	149	151	15	13	2

Table 1: Example failures of conjecture 3[3]

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2 Numerical Check

Using GPT-6 Astra for coding, we run a numerical check across 1,325,192 polynomials using the proxy $D_r(x)$ as defined in their paper

Number of prime factors	Each prime below	Polynomials checked
2	200	990
3	1,000	762,355
4	200	148,995
5	150	278,256
6	100	134,596

Table 2: Numerical search space

We find 2,771,344 counterexamples to conjecture 3[3] as discussed previously.
No counterexamples to conjecture 3[1] and 3[2] were found in our search. .

3 An Observation

Hong and Sommesse propose the following conjecture.

Conjecture 3.

$$N_n = 2k - 1 \quad \text{if } n \text{ is the product of } k \text{ smallest odd primes}$$

We find support for two statements that, together, imply Hong and Sommesse's Conjecture.

We reintroduce further notation from the author's original paper. Let $m = p_1 \cdots p_k$ be a product of k distinct odd primes. Furthermore, let N_m^+ and N_m^- for the numbers of positive and negative real zeros of Φ'_m (counted with multiplicity).

Conjecture For every such m ,

$$N_m^+ \geq k - 1, \quad N_m^- \geq k$$

These conjectures both hold for the 1,326,776 tested cases and, together, imply conjecture 3[2].

References

- [1] H. Hong and A. J. Sommesse, *Number of Real Critical Points of Cyclotomic Polynomials*, arXiv:1912.12253 (2019). <https://arxiv.org/abs/1912.12253>.